St Catherine's
School
Waverley, Sydney

Student Number:..

Year 12

Extension 1 mathematics-Trial HSC examination

3 August 2011

Time allowed: 2 hours

Reading time: 5 minutes

Course weighting: 40%

General Instructions

- Attempt ALL questions
- Write your Student NUMBER on every booklet used.
- Write using blue or black pen
- Board approved calculators and stencils can be used
- A table of standard integrals is provided
- Show necessary working
- Organise Q 1 to 4 in one bundle and Q 4 to 7 in another

Attempt Questions 1 to 7-All questions are of equal value

Total marks /84

Question 1

a) Use the result $\lim_{x\to 0}\frac{\sin x}{x} = 1$ to find the value of $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$ 2m

b) (i) Show that the curves $y = \log_e x$ and $xy = e$ meet at the point (e,1). 1m

(ii) If α is the acute angle between the tangents at the point of intersection show that 3m

$$tan\alpha = \frac{2e}{e^2-1}$$

c) Eight people are seated around a table.

(i) How many arrangements are possible if there are no restrictions? 1m

(ii) Laura, Sally and Lauren want to sit together. How many arrangements are possible? 2m

d) Integrate using the substitution $u = 1 - x$

$$\int \frac{x}{\sqrt{1-x}}dx$$ 3m

Question 2

Start a new page

a) Evaluate and leave in exact form. **3m**

$$\int_0^{\frac{3}{2}} \frac{dx}{\sqrt{9-2x^2}}$$

b) P(x) is an even polynomial of degree 4 .i.e. $P(-x) = P(x)$.

Two of its zeros are at $x = 1$ *and* $x = 2$

(i) Find the other two roots. **2m**

(ii) If P(0)=8, Write $P(x)$ in factored form. **2m**

c) (i) Show that $\sqrt{3}\cos x - \sin x = 2\cos(x + \frac{\pi}{6})$ **1m**

(ii) Sketch the function $y = \sqrt{3}\cos x - \sin x$ in the domain $0 \le x \le 2\pi$ clearly labelling the end points and the turning points. **2m**

(iii) Find the general solution to the equation **2m**

$$\sqrt{3}\cos x - \sin x = 1$$

Question 3

Start a new page

a) (i) Write the expansion of $(5 + 2x)^{15}$ in ascending powers of x and show that

$$\frac{coefficent\ of t_{r+1}}{coefficient\ of\ t_r} = \frac{32-2r}{5r}$$ **3m**

where t_r is the rth term in the above expansion.

(ii) Hence find the greatest coefficient **2m**

(You may leave the answer in terms of the binomial coefficient.)

b) Find the constant term in the expansion of **3m**

$$\left(x - \frac{1}{2x^3}\right)^{12}$$

You may leave the answer in terms of the binomial coefficient.

c) A machine is known to produce items of which 3% are too short, 7% are too long and 90% are satisfactory. A random sample of twelve items is taken from the sample.

Find the probability (correct to two decimal places) that

(i) At most one of these items is too long **2m**

(ii) At least ten of these items are satisfactory. **2m**

Question 4

Start a new page

a) Consider the function
$$f(x) = \frac{e^x}{x-1}$$

(i) Show that $(2, e^2)$ is a minimum turning point. **3m**

(ii) Determine all the vertical and horizontal asymptotes of the curve y=f(x) and sketch the graph y=f(x) including any intercepts with the coordinate axes. **3m**

b) Let $f(x) = 4x - x^2$ for $x \geq 2$.

(i) Sketch the function $y = f(x)$ *and* $y = f^{-1}(x)$ *on the same set of axes.* **2m**

(ii) Find an expression for $y = f^{-1}(x)$ **2m**

(iii) Find the point of intersection of $y = f(x)$ *and* $y = f^{-1}(x)$ **2m**

Question 5

START A NEW BOOKLET

a) Consider the parabola $x^2 = 4ay$. Let S *(0,a)* be the focus.
The tangents at points P: *(2ap,ap²)* and Q: *(2aq,aq²)* meet at the point T.

(i) Show that the coordinates of T is *(a(p+q) , apq)*
You may assume that the equation of the tangent at any point P : *(2ap,ap²)* is $y = px - ap^2$ **2m**

(ii) Show that $SP = a(p^2 + 1)$ (Draw a figure) **1m**

(iii) P and Q move on the parabola so that $SP + SQ = 4a$
Find the locus of T. **2m**

b) A particle is moving in a straight line in Simple Harmonic Motion according to the equation **3m**
$$\frac{d^2y}{dx^2} = -x$$
Initially the particle is at $x = 1$ and has a velocity of 1cm /sec.
Show that $x = \sqrt{2}\cos(t \overset{-}{\cancel{+}} \frac{\pi}{4})$

c) Suppose that the cubic function $f(x) = x^3 + ax^2 + bx + c$ has a relative maximum at $x = \alpha$ and a relative minimum at $x = \beta$.

(i) Prove that $\alpha + \beta = \frac{-2a}{3}$ **2m**

(ii) Deduce that the point of inflexion occurs at
$$x = \frac{\alpha + \beta}{2}$$ **2m**

Question 6

Start a new page

a) Let $f(x) = \frac{1}{2}\cos^{-1}(1-3x)$

(i) Find the domain and the range — 2m

(ii) Sketch the function clearly — 1m

b) Integrate using the substitution $x = 3\sin\theta$ — 4m

$$\int_0^3 \frac{x^2\,dx}{\sqrt{9-x^2}}$$

c) If $f(x) = \cos^{-1}(\sin x)$

(i) Show that $f'(x) = \pm 1$. (note that $\sqrt{x^2}=|x|$) — 3m

State the values of x for which it is 1 or -1 in the domain

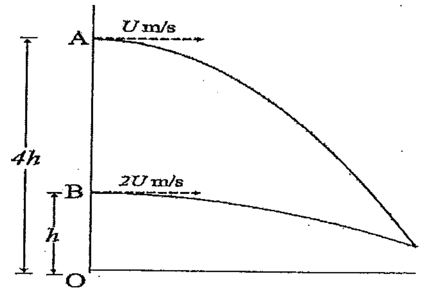
$-\pi \le x \le \pi$

(ii) Hence sketch the function $f(x) = \cos^{-1}(\sin x)$ in the domain $-\pi \le x \le \pi$ — 2m

Question 7

Start a new page

a) Fig



A vertical building stands with its base O on the horizontal ground. A and B are two points on the building vertically above each other such that A is 4h metres above O and B is h metres above O.

A particle is projected horizontally from A with a speed of U m/sec and 10 seconds later a second particle is projected horizontally from B with a velocity of 2U m/sec. The two particles collide t seconds after the first particle is projected.

The coordinate axes are placed at O and the value of g is taken as 10 m/sec^2

(i) Show that the expressions for the horizontal and vertical displacements of each particle, t seconds after the first particle is projected are given by

$x_A = Ut \text{ and } y_A = 4h - 5t^2$ for the particle A

and

$x_B = 2U(t-10) \text{ and } y_B = h - 5(t-10)^2$ for the particle B. — 4m

(ii) Find the time taken for the particles to collide — 1m

(iii) Find the value of h. — 2m

Please turn over for 7(b)

b) (i) Show that

$$(1-x)^n\left(1+\frac{1}{x}\right)^n = \frac{(1-x^2)^n}{x^n}$$ 1m

(ii) Show that

$$\frac{(1-x^2)^n}{x^n} = \sum_{0}^{n}(-1)^r\binom{n}{r}x^{2r-n}$$ 1m

(note: $\binom{n}{r} = {}^{n}C_{r}$)

3m

(iii) By considering the coefficient of x^2 on both sides of the given identity in part (i) and using part (ii), show that

$$\binom{n}{2}\binom{n}{0} - \binom{n}{3}\binom{n}{1} + \cdots \ldots + (-1)^n\binom{n}{n}\binom{n}{n-2}$$

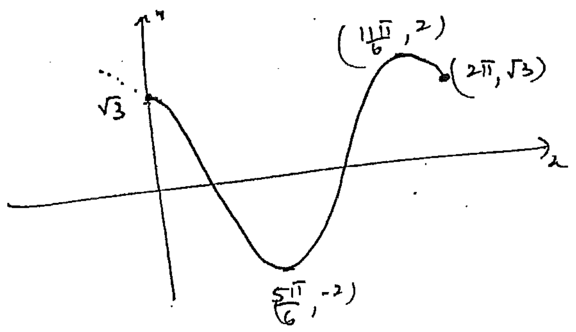
$$= \begin{cases} 0, \text{ if } n \text{ is odd} \\ (-1)^{\frac{n+2}{2}}\binom{n}{\frac{n+2}{2}} \text{ if } n \text{ is even} \end{cases}$$

END OF Paper

HSC Maths [illegible] – Ext. 1 Mathematics

Qn	Solutions	Marks	Comments: Criteria
1.	$\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$		
	$= \lim_{x\to 0} \frac{1-(1-2\sin^2 x)}{x^2}$	(1m)	
	$= \lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)^2$		
	$= 2\left(\lim_{x\to 0} \frac{\sin x}{x}\right)^2$	(1m)	
	$= 2 \times 1 = 2.$		
b)	$(e,1)$ lies on $y = \log_e x$ for $1 = \log_e e$.	(1m)	
	also " " $xy = e$ for $e(1) = e$		
	$y = \log_e x$; $y'_{\text{at }(e,1)} = \frac{1}{e}$	$\frac{1}{2}$	
	$y' = \frac{1}{x}$		
	$y = \frac{e}{x}$; $y'_{\text{at }(e,1)} = \frac{-e}{e^2} = \frac{-1}{e}$	$\frac{1}{2}$	
	$y' = -\frac{e}{x^2}$		
	if α is the acute angle between the tangents		
	$\tan\alpha = \left\lvert \frac{\frac{1}{e} - \frac{-1}{e}}{1 - \frac{1}{e^2}} \right\rvert = \left\lvert \frac{2/e}{e^2-1/e^2} \right\rvert$	$= \left\lvert \frac{2e}{e^2-1} \right\rvert$	(1) for correct substitution
		$= \frac{2e}{e^2-1}$	(1) to simplify.
c)	(i) $7!$	(1)	
	(ii) $5! \times 3!$		
	(1) (1)		

Qn	Solutions	Marks	Comments: Criteria
d)	$u = 1-x$		
	$du = -dx$		
	$\int \frac{x\,dx}{\sqrt{1-x}} = \int \frac{(1-u)(-du)}{\sqrt{u}}$	(1m)	
	$= \int \left(\sqrt{u} - u^{-1/2}\right) du$	($\frac{1}{2}$m)	
	$= \frac{2}{3}u^{3/2} - 2u^{1/2} + c$	(1m)	
	$= \frac{2}{3}(1-x)^{3/2} - 2\sqrt{1-x} + c.$	($\frac{1}{2}$m)	
2	$\int_0^{3/2} \frac{dx}{\sqrt{9-2x^2}} = \int_0^{3/2} \frac{dx}{\sqrt{2\left(\frac{9}{2}-x^2\right)}}$	(1m)	
	$= \frac{1}{\sqrt{2}} \cdot \sin^{-1}\left(\frac{\sqrt{2}x}{3}\right)_0^{3/2}$	(1m)	
	$= \frac{1}{\sqrt{2}} \cdot \left(\sin^{-1} \frac{\sqrt{2}}{3}\cdot\frac{3}{2} - \sin^{-1} 0\right)$		
	$= \frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}} = \frac{\sqrt{2}\pi}{8}$	(1m)	full mark at this point
b)	$P(-x) = P(x)$ (for P is a zero-given)		
	$\therefore P(-1) = P(1) = 0$.		
	$P(-2) = P(2) = 0$.		
	(i) The other two roots are -1 and -2		
	(ii) $P(x) = A(x-1)(x+1)(x-2)(x+2)$		
	$P(0) = 8$		
	$\therefore 8 = A(-1)(-4)$		

Qn	Solutions	Marks	Comments: Criteria
	$\therefore A = 2$		
	$P(x) = 2(x-1)(x+1)(x-2)(x+2)$		
(i)	$\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$		
	$= R(\cos x\cos\alpha - \sin x\sin\alpha)$		
	$\therefore \sqrt{3} = R\cos\alpha$ $\quad \therefore \tan\alpha = \frac{1}{\sqrt{3}}$		
	$1 = R\sin\alpha$ $\quad \alpha = \frac{\pi}{6}$ (1)		
	& $R = 2$ (1)		
(ii)			end pts. 1m. turning pts 1m.
(iii)	$2\cos\left(x+\frac{\pi}{6}\right) = 1$	($\frac{1}{2}$)	
	$\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$	($\frac{1}{2}$)	
	$x+\frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$	(1)	← full marks awarded at this point.
	$x = 2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}$		
	$x = 2n\pi + \pi/6$ or $2n\pi - \frac{\pi}{2}$		

Qn	Solutions	Marks	Comments: Criteria
3	$(5+2x)^{15} = {}^{15}C_0\, 5^{15} + {}^{15}C_1 \cdot 5^{14}(2x) + \dots$		
	$t_{r+1} = {}^{15}C_r \cdot 5^{15-r} \cdot (2x)^r$	(1m)	
	$t_r = {}^{15}C_{r-1} \cdot 5^{15-(r-1)} \cdot (2x)^{r-1}$		
	$\frac{\text{Coeff of } t_{r+1}}{\text{Coeff of } t_r} = \frac{{}^{15}C_r}{{}^{15}C_{r-1}} \cdot \frac{5^{15-r}}{5^{16-r}} \cdot \frac{2^r}{2^{r-1}}$	(1m)	
	$= \frac{15!}{r!(15-r)!} \cdot \frac{(r-1)!(15-(r-1))!}{15!} \cdot \frac{2}{5}$		
	$= \frac{16-r}{r} \cdot \frac{2}{5} = \frac{32-2r}{5r}$	(1m)	
(ii)	Coeff of $t_{r+1} \geq$ Coeff of t_r		
	when $32-2r \geq 5r$ $\quad (5r\dots)$		
	$7r \leq 32$	(1m)	
	i.e. when $r = 1, 2, 3, 4$		
	Coeff of $t_{r+1} <$ Coeff of t_r		
	for $r = 5, 6, \dots$		
	$\therefore t_5$ has the highest coeff.	(1m)	
	Coeff of $t_5 = {}^{15}C_4 \cdot 5^{11} \cdot 2^4$.		

Qn	Solutions	Marks	Comments: Criteria
b)	$\left(x - \frac{1}{2x^3}\right)^{12} = {}^{12}C_0\, x^{12} - {}^{12}C_1\, x^{11}\left(\frac{1}{2x^3}\right) + \cdots$		
	$t_{r+1} = {}^{12}C_r\, x^{12-r}\left(-\frac{1}{2}\right)^r \cdot x^{-3r}$		
	$= \left(-\frac{1}{2}\right)^r \cdot {}^{12}C_r \cdot x^{12-4r}$	(1)	
	for Const. term; $12-4r=0$, $\underline{r=3}$	(1)	
	Const. term is $\left(-\frac{1}{2}\right)^3 \cdot {}^{12}C_3$.	(1)	
c)	$P(\text{too long}) = 0.07 = p$ (say) $P(\text{not too long}) = 0.93 = q$ (say) $q = 1-p$	1m	
	Consider $(p+q)^{12} = {}^{12}C_0\, p^{12} + {}^{12}C_1\, p^{11} q + \cdots$		
	$P(\text{at most one too long})$		1m
	$= P(\text{none too long}) + P(\text{one too long})$		
	$= {}^{12}C_{12}\, q^{12} + {}^{12}C_{11}\, p\, q^{11}$		
	$= (0.93)^{12} + 12(0.07)(0.93)^{11}$	1m	
	$= 0.80$.		
(ii)	$P(\text{satis}) = 0.9 = p$ (say) $P(\text{not satis}) = 0.1 = q$ (say); $q = 1-p$.		1m

Qn	Solutions	Marks	Comments: Criteria
	$(p+q)^{12} = {}^{12}C_0\, p^{12} + {}^{12}C_1\, p^{11} q + \cdots$		
	$P(\text{atleast 10 satisf})$ $= P(\text{all satis}) + P(\text{11 satis}) + P(\text{10 satis})$		
	$= {}^{12}C_0\, p^{12} + {}^{12}C_1\, p^{11} q + {}^{12}C_2\, p^{10} q^2$		
	$= {}^{12}C_0 (0.9)^{12} + {}^{12}C_1 (0.9)^{11}(0.1) + {}^{12}C_2 (0.9)^{10}(0.1)^2$		
	$= \cancel{0.93}$ 0.89 (2 d.p.)		
Q.4	$f(x) = \frac{e^x}{x-1}$		
	$f'(x) = \frac{(x-1)e^x - e^x(1)}{(x-1)^2}$		
	$= \frac{e^x(x-2)}{(x-1)^2}$	(1m)	
	Stat. pts; $f'(x) = 0$ $e^x(x-2) = 0$ $\therefore x = 2$ $e^x \neq 0$		
	$y = \frac{e^2}{2-1}$ $\therefore (2, e^2)$ is a stat. pt.		(1m)
	$f'(1.9) < 0$; $f'(2.1) > 0$	✓	
	$\therefore (2, e^2)$ is a min. t. pt.		(1m)

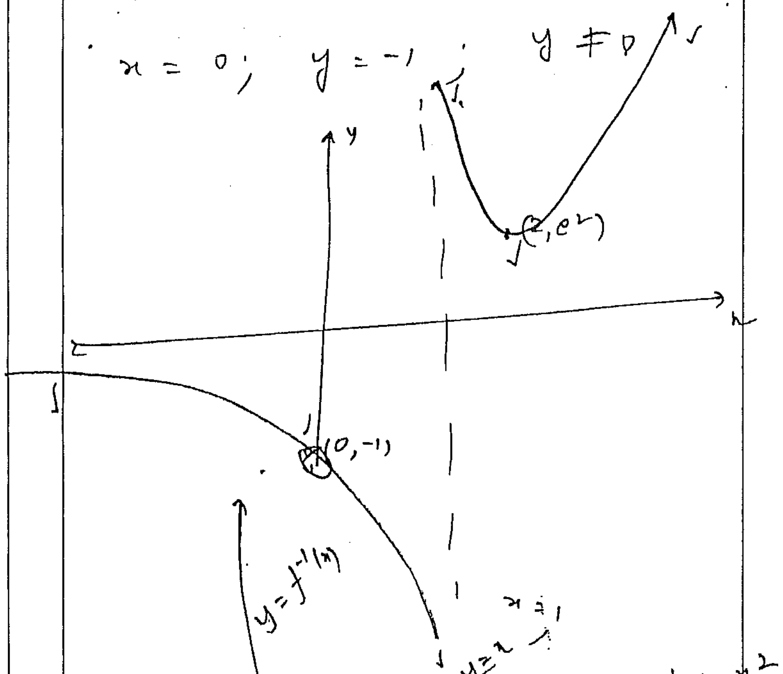
Qn	Solutions	Marks	Comments: Criteria

11) Vertical asym $x = 1$

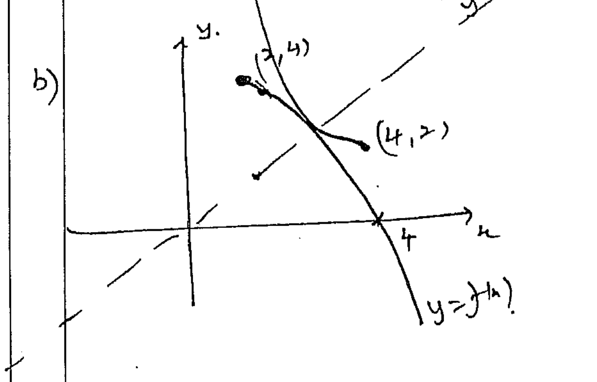
$\lim_{x \to \infty} f(x) = \infty$ Horiz asym $y = 0$

$\lim_{x \to -\infty} f(x) = 0$.

$x = 0;\ y = -1$; $y \neq 0$ ✓



b)



$y = 4x - x^2$
$= x(4-x)$
$x = 2;\ y = 4$
$(2,4)$ is turning pt

Qn	Solutions	Marks	Comments: Criteria

(ii) $y = 4x - x^2$
$x = 4y - y^2$ (inv. fn)

$y^2 - 4y + x = 0$

$y = \dfrac{4 \pm \sqrt{16 - 4x}}{2}$

$= \dfrac{4 \pm 2\sqrt{4-x}}{2}$

Range of $y = f^{-1}(x)$ = Domain of $y = f(x)$
≥ 2

$\therefore\ y = 2 + \sqrt{4-x}$.

(iii) $y = f(x)$ and $y = f^{-1}(x)$ meet on $y = x$

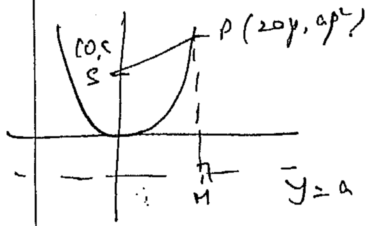
$\therefore$ Their pt. of intersection is given by $4x - x^2 = x$

$3x - x^2 = 0$

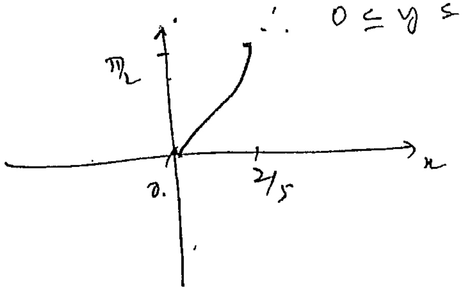
$x = 0$; $x = 3$

$x = 0$ is not in the domain.

$\therefore (3, 3)$ is the pt. of intersection

Qn	Solutions	Marks	Comments: Criteria
5.	$x^2 = 4ay$.		
	Eqn of the tgt at $P: (2ap, ap^2)$		
	$y = px - ap^2$ — (1)		
	Eqn of the tgt at Q is		
	$y = qx - aq^2$ — (2)		
	(1) & (2) meet at		
	$px - ap^2 = qx - aq^2$		
	$(p-q)x = a(p^2-q^2)$		
	$x = a(p+q)$ $(p \neq q)$		
(ii)	Sub in (1) $y = pa(p+q) - ap^2$		
	$= apq$.	(1m)	
			
	$SP = PM$ by defn.		
	$= ap^2 + a$		
	$= a(p^2+1)$	(1m)	
(iii)	$SP + SQ = 4a$		
	$a(p^2+1) + a(q^2+1) = 4a$		
	$\therefore p^2 + q^2 = 2$	$(\frac{1}{2}m)$	attempt to combine $\frac{1}{2}m$.
	$\therefore T$ moves so that $x = a(p+q)$		
	$y = apq$ and $p^2+q^2 = 2$	(1m)	
	$(p+q)^2 = p^2+q^2+2pq$		
	$\left(\frac{x}{a}\right)^2 = 2 + 2apq$ is locus		

Qn	Solutions	Marks	Comments: Criteria
b)	Let $x = a\cos(nt+\epsilon)$.		
	$n=1$		
	at $t=0$; $x=1$		
	$\therefore 1 = a\cos\epsilon$ — (1)		
	$\dot{x} = -an\sin(nt+\epsilon)$		
	$n=1$, $t=0$, $\dot{x}=1$ $\therefore 1 = -a\sin\epsilon$ — (2)		
	$\therefore \tan\epsilon = -1$ & $a\cos(-\pi/4) = 1$		
	$\epsilon = -\pi/4$ $\therefore a = \sqrt{2}$		
	$\therefore x = \sqrt{2}\cos(t - \pi/4)$		
c)	$f(x) = x^3 + ax^2 + bx + c$		
	$f'(\alpha) = 0$; $f'(\beta) = 0$		
	$\therefore \alpha$ & β are roots of		
	$f'(x) = 3x^2 + 2ax + b = 0$		
	$\therefore \alpha + \beta = \frac{-2a}{3}$.	(2m)	
	$f''(x) = 6x + 2a$.		
	possible pt. of inflexion is when $f''(x) = 0$; $6x + 2a = 0$		
	$x = -\frac{2a}{6}$		
	$= -(\alpha+\beta) \times -\frac{1}{2}$		
	$= \frac{\alpha+\beta}{2}$.	(1m)	

Qn	Solutions	Marks	Comments: Criteria
	$\frac{\alpha+\beta}{2}$ is between α & β		
	$f''(\alpha)<0$; $f''\left(\frac{\alpha+\beta}{2}\right)=0$; $f''(\beta)>0$	(1m)	
	$\therefore\ x=\frac{\alpha+\beta}{2}$ is a pt. of inflexion		
Q.6	$f(x) = \frac{1}{2}\cos^{-1}(1-3x)$		
	Dom: $-1 \le 1-3x \le 1$		
	$-2 \le -3x \le 0$		
	$0 \le x \le \frac{2}{3}$.		
	Range: when $x=0$; $f(0) = \frac{1}{2}\cos^{-1} 1 = 0$		
	when $x=\frac{2}{3}$; $f\left(\frac{2}{3}\right) = \frac{1}{2}\cos^{-1}(1-2) = \frac{\pi}{2}$.		
	$\therefore\ 0 \le y \le \pi/2$		
			

Qn	Solutions	Marks	Comments: Criteria
b)	$\int_0^3 \frac{x^2\,dx}{\sqrt{9-x^2}}$		
	Let $x = 3\sin\theta$		
	$dx = 3\cos\theta\,d\theta$		
	$x=0$; $\theta=0$		
	$x=3$; $\theta=\pi/2$.		
	$\therefore \int_0^3 \frac{x^2\,dx}{\sqrt{9-x^2}}$		
	$= \int_0^{\pi/2} \frac{9\sin^2\theta \cdot 3\cos\theta}{\sqrt{9-9\sin^2\theta}}\,d\theta$	(1m)	
		½(m)	
	$= 9\int_0^{\pi/2} \sin^2\theta\,d\theta$	(1m)	
	$= 9\int_0^{\pi/2} \left(\frac{1-\cos 2\theta}{2}\right)$	(1m)	
	$= \frac{9}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi/2}$	½.	
	$= \frac{9}{2}\cdot\frac{\pi}{2} = 9\pi/4$		

Qn	Solutions	Marks	Comments: Criteria

c)

$$f(x) = \cos^{-1}(\sin x)$$

$$f'(x) = \frac{-1}{\sqrt{1-\sin^2 x}} \cdot \cos x$$

$$= \frac{\cos x}{|\cos x|}$$ ($1\frac{1}{2}$m)

$= -1$ when $\cos x > 0$ ($\frac{1}{2}$m)

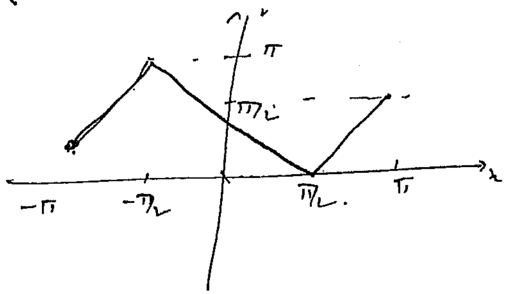
$= 1$ when $\cos x < 0$

$f'(x) = -1$; when $\frac{-\pi}{2} < x < \pi/2$

$f'(x) = 1$ when $\frac{\pi}{2} < x < \pi$ or $-\pi < x < -\pi/2$ } 1m

$f(-\frac{\pi}{2}) = \pi$; $f(0) = \pi/2$; $f(\pi/2) = 0$

$f(-\pi) = \pi/2$; $f(\pi) = \pi/2$



Qn	Solutions	Marks	Comments: Criteria

7 (A)

$\ddot{x} = 0$

$\dot{x} = \text{const.}$

at $t=0$; $\dot{x} = u$

$\therefore \dot{x} = u$

$x = ut + c$

$t=0$; $x = 0$

$\therefore x = ut$

$\ddot{y} = -10$

$\dot{y} = -10t + c$

$t=0$; $\dot{y} = 0 \therefore c = 0$

$\dot{y} = -10t$

$y = -5t^2 + c$ $\begin{cases} t = 0 \\ y = 4h \end{cases}$

$\therefore c = 4h$

$\therefore y = 4h - 5t^2$.

(B)

$\ddot{x} = 0$

$\dot{x} = \text{const.}$

$t = 10$; $\dot{x} = 2u$

$\therefore \dot{x} = 2u$.

$x = 2ut + C$

at $t = 10$; $x = 0$

$0 = 20u + c$

$\therefore x = 2ut - 20u$

$= 2u(t-10)$

$\ddot{y} = -10$

$\dot{y} = -10t + c$

$t = 10$; $\dot{y} = 0$.

$\therefore c = 100$

$\dot{y} = -10t + 100$

$y = -5t^2 + 100t + c$

$t = 10$; $y = h$

$\therefore h = -500 + 1000 + c$

$\therefore c = h - 500$

$\therefore y = -5t^2 + 100t + h - 500$

$= h - 5(t^2 - 20t + 100)$

$= h - 5(t-10)^2$

Qn	Solutions	Marks	Comments: Criteria
	where they collide		
	$x_A = x_B$; $\quad ut = 2ut - 20u$		
	$\therefore t = 20$ sec		
	$y_A = y_B$; $\quad 4h - 5t^2 = h - 5(t-10)^2$		
	$3h = 5(t^2 - (t-10)^2)$		
	$t = 20$		
	$3h = 1500$		
	$h = 500$ m.		
b)	$(1-x)^n (1+\frac{1}{x})^n$		
	$= (1-x)^n \frac{(x+1)^n}{x^n}$		
	$= \frac{(1-x^2)^n}{x^n}$		
(ii)	$\frac{(1-x^2)^n}{x^n} = \frac{1}{x^n}({}^nC_0 - {}^nC_1 x^2 + {}^nC_2 x^4 \ldots\ldots)$		
	$= \frac{1}{x^n} \sum_0^n {}^nC_r (-1)^r x^{2r}$		
	$= \sum_0^n (-1)^r \, {}^nC_r \, x^{2r-n}$.		

Qn	Solutions	Marks	Comments: Criteria
	Coeff of x^2 in		
	$\sum_0^n (-1)^r \, {}^nC_r \, x^{2r-n}$ is when		
	$2r - n = 2$ or when $r = \frac{n+2}{2}$		
	$\therefore$ Coeff is $(-1)^{\frac{n+2}{2}} \, {}^nC_{\frac{n+2}{2}}$		
	This is zero when n is odd; for $\frac{n+2}{2}$ will not be a whole number		
	$\therefore (-1)^{\frac{n+2}{n}} \cdot {}^nC_{\frac{n+2}{2}}$ is the Coeff of x^2 when n is even.		
	LHS		
	$({}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 \ldots\ldots)({}^nC_0 + {}^nC_1 \cdot \frac{1}{x} + {}^nC_2 \cdot \frac{1}{x^2} + \ldots\ldots)$		
	Coeff of x^2:		
	$({}^nC_2 \cdot {}^nC_0) - ({}^nC_3 \cdot {}^nC_1) + ({}^nC_4 \cdot {}^nC_2) \ldots + (-1)^n \cdot {}^nC_n \cdot {}^nC_{n-2}$.		
	Hence the result.		